TABLE II
TE CUTOFF WAVENUMBERS OF ECCENTRIC WAVEGUIDES FOR VARIOUS η AND D

	SYMMETRIC			ANTISYMMETRIC		
	Present Method	Lower Bound	Upper Bound	Present Method	Lower Bound	Upper Bound
η = 0.5, d = 0.1	5.4695	5.46911	5.4704	5.9918	5.99121	5.99251
	6.4747	6.47403	6.47547	6.9203	6.91953	6.92102
	7.3062	7.30527	7.30683	7.7123	7.71130	7.71299
	7.8692	7.86823	7.86982	8.4845	8.4830	8.4857
	8.4965	8.4950	8.4972	9.3564	9.3542	9.3572
η = 0.5, d = 0.2	4.8106	4.80953	4.81191	5.5114	5.5098	5.5125
	6.1724	6.1703	6.1735	6.7991	6.7964	6.8002
	7.3945	7.3907	7.3957	7.9607	7.9559	7.9619
	8.4974	8.4894	8.4991	9.0091	8.9996	9.0106
	9.3409	9.2694	9.3485	9.9556	9.9316	9.9677
η = 0.5, d = 0.3	4.3071	4.3042	4.3118	5.1222	5.1179	5.1257
	5.8903	5.8730	5.8940	6.6210	6.5994	6.6251
	7.3197	7.240	7.325	7.9910	7.878	7.997
	8.2909	8.081	8.316	9.1877	8.829	9.210
	8.6388	8.382	8.648	9.2676	8.900	9.276
η = 2/3, d = 0.2	6.2399	6.2379	6.2429	6.9683	6.9654	6.9702
	7.6769	7.6728	7.6787	8.3682	8.3631	8.3800
	9.0439	9.0323	9.0456	9.7053	9.6922	9.7071
	10.3536	10.318	10.356	10.9892	10.947	10.992
	11.6184*	11.539	11.616	12.2266*	12.128	12.219
η = 0.25, d = 0.25	3.4723	3.4687	3.4752	4.2640	4.2583	4.2680
	4.9221	4.9110	4.9249	5.5393	5.5239	5.5423
	5.9268	5.393	5.932	6.6357	6.502	6.941
	6.7154	6.591	6.723	7.7135	7.443	7.723
	6.7527	6.622	6.767	7.7243	7.488	7.735
η = 0.25, d = 0.5	2.9824	2.887	2.996	4.0338	3.858	4.043
	4.7868	4.088	4.827	5.5432	4.50	5.75
	5.8084		5.977	6.9144		6.992
	6.2439		6.323	7.1560		7.208
	7.5592		7.735	8.1858		8.395

Lower and upper bounds are from [6].

allow for comparison with the results previously published by Kuttler [6].

Table I presents the symmetric and antisymmetric cutoff wavenumbers in terms of η and d for TE modes, while the corresponding values for TM modes appear in Table II. Both tables also list the lower and upper bounds provided by Kuttler [6]. In most cases, the actual cutoff wavenumber lies close to Kuttler's upper bound, while in some cases (marked by an asterisk), they fall outside of the bounds. Furthermore, in three situations marked by a bar (—), it was not possible to obtain a zero of the determinant within or near the ranges reported by Kuttler, even when taking determinants of order 15.

All the calculations were made with perfect conductor boundaries, i.e., the effect of metallic losses was not taken into account. As the values of the fields can be determined, the effect of finite metal conductivity on the cutoff may be evaluated using the usual perturbation technique. It should, however, be noted that the cutoff wavenumber then becomes complex, so that the cutoff is not clearly defined as in the lossless case.

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Fully Computer-Aided Synthesis of a Planar Circulator

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I. INTRODUCTION

A planar junction circulator consists, in general, of a three-fold symmetric resonator of arbitrary shape to which three transmission lines are connected. So far, the circulators having disk [1], triangle, and hexagonal resonators [2], [3] have been studied in detail. In the analysis of the circuit parameters of circulators, two general methods which were presented in 1977 [4] are used widely. One is based upon a contour-integral solution of the wave equation. In the other approach, the circuit parameters of the junction are expanded in terms of the eigenmodes of the magnetized planar resonator. Since both methods have been applied successfully to various circulators, there is no doubt as to the usefulness of the methods at present.

As a next step, we have to develop an algorithm to synthesize a circulator. We had studied the optimum design of a planar circulator for wide-band operation based upon a computer-aided, but trial-and-error, approach [5]. The results obtained were far

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from being general. In this paper, a fully computer-aided synthesis of a planar circulator is discussed. We take the following method of synthesis. First, a ferrite resonator with three-fold symmetry and an arbitrary shape is described by several variables. The frequency dependence of the initial planar circulator is analyzed by the contour-integral method, and the evaluation function which evaluates the perfect circulation condition and the pattern simplicity is computed by using the computed circuit parameters. Next, the variables are adjusted to decrease the evaluation function according to an algorithm. These processes are repeated until the evaluation function decreases to a certain number. As a result, we succeeded in synthesizing an arbitrarily-shaped planar circulator better than a six-sided (irregular hexagonal) circulator.

II. Method of Synthesis

In the synthesis of a circulator, the analysis of frequency dependence, the evaluation of circulator performance, and the deformation of variables are important processes. Each process will be explained in brief in this section. Because some of the mathematical methods used are well established, we will explain only the methods peculiar to our synthesis.

A. Analysis of Frequency Dependence

The model of a planar circulator used in the synthesis is shown in Fig. 1(a). The contour-integral method used in the analysis has been modified to improve the computational efficiency since the first presentation. One is based upon the fact that the Green's function in the integral equation method is not unique [6]. The use of the symmetry of a circuit is also a considerable improvement of the method to reduce the dimensions of the matrices to be inverted in the computation of circuit parameters [2]. In this paper, such an improved integral equation method is used for the analysis of the frequency dependence of the circulator.

B. Evaluation Function

The evaluation function used in the synthesis evaluates the two figures of merit. One is related to the circulation condition and the other evaluates the simplicity of the pattern.

1) Evaluation of Circulation Condition: To evaluate the circulator performance in a wide frequency range, the following evaluation function F_1 is used in this paper:

$$F_1 = \sum_{i=1}^{M} \{ a_i |g(f_i) - 1| + b_i |y(f_i)| \}. \tag{1}$$

Here, f_i $(i = 1, \cdots, M)$ are the sampling frequencies only where the circulator performance is evaluated, and a_i and b_i are the weighting parameters. $g(f_i) + jy(f_i)$, which is computed at each sampling frequency, is defined as

$$g = -j\frac{\sqrt{3}}{2}\left(\frac{y_1 y_3 - 1}{y_1 - y_3} - \frac{y_1 y_2 - 1}{y_1 - y_2}\right) \tag{2}$$

$$y = -j\frac{1}{2}\left(\frac{y_1 y_3 - 1}{y_1 - y_3} + \frac{y_1 y_2 - 1}{y_1 - y_2}\right) \tag{3}$$

where y_1 is the normalized eigenadmittance for the in-phase excitation and y_2 and y_3 are for the rotational excitation. They are obtained from the computed elements of the impedance matrix (see [5, eq. (10)]). Since $g + jy = 1 + j0$ is equivalent to the following perfect circulation conditions

$$y_2 = \frac{y_1 + j\sqrt{3}}{1 + j\sqrt{3}\, y_1} \qquad y_3 = \frac{y_1 - j\sqrt{3}}{1 + j\sqrt{3}\, y_1} \tag{4}$$

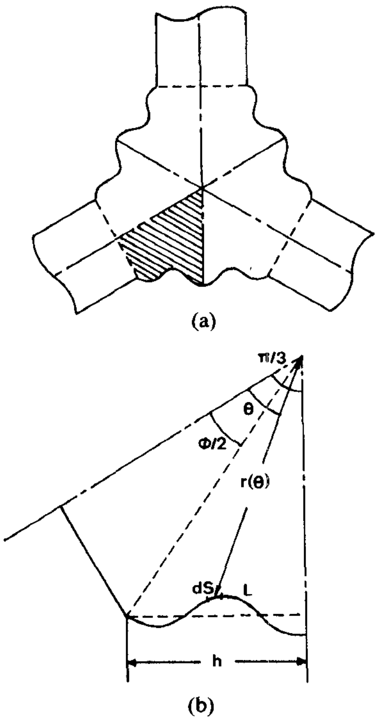

Fig. 1. (a) Planar circulator with a three-fold symmetric resonator. (b) One-sixth portion of a circulator used in the synthesis.

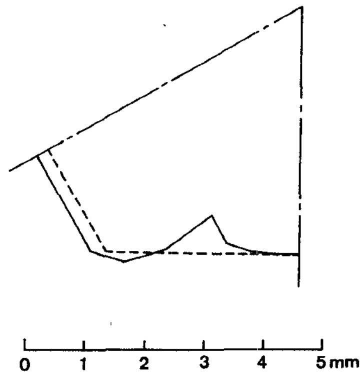

Fig. 2. Synthesized circulator pattern (solid lines) and initial six-sided circulator after the preliminary synthesis (dashed lines).

the evaluation function defined in (1) will approach zero when the circulation conditions are satisfied in a wide frequency range between f_1 and f_M.

2) Evaluation of Pattern Simplicity: We included, also, an additional evaluation function F_2 to realize a relatively simple circulator pattern. This simplification, based on our accumulated experience, is important because the circulator characteristics are sensitive to several changes of the contour and because it is needed to make the fabrication of the center conductor practically possible. Considering the threefold symmetry of a circulator, only one-sixth of the area of the circulator is synthesized. This is shown as a cross-hatched portion in Fig. 1(a). Note that the whole circuit can be analyzed by using the parameters describing a one-sixth portion of the circulator.

To evaluate effects of this pattern simplification, we introduced the following second evaluation function defined by:

$$F_2 = c(L/h - 1) \tag{5}$$

where c is a weighting parameter for F_2 and L is the total length

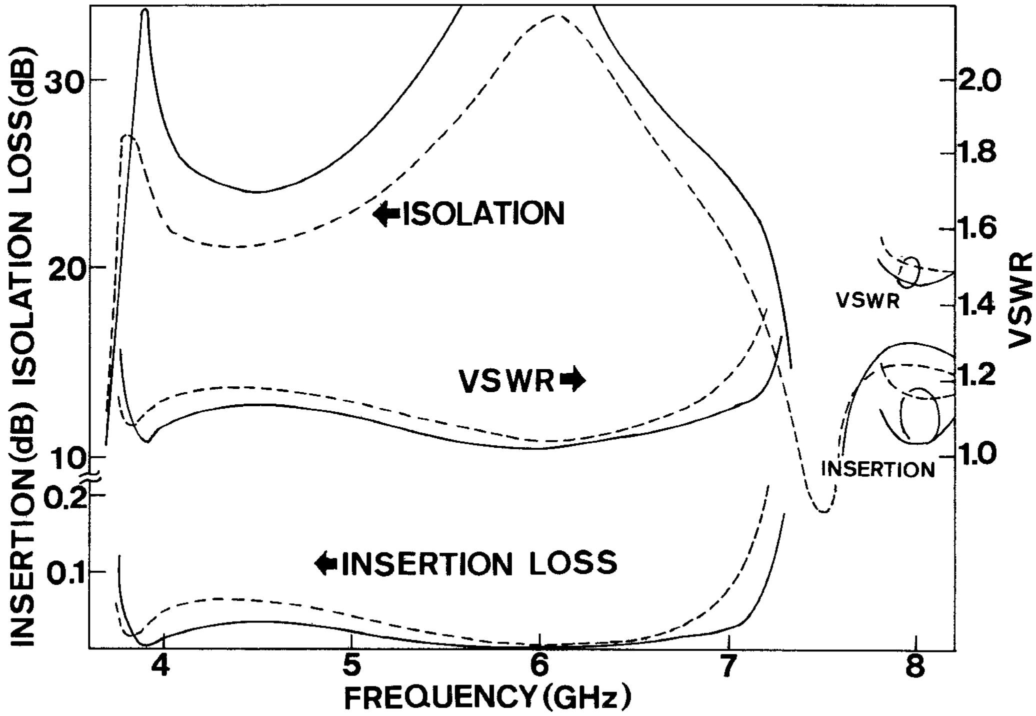

Fig. 3. Computed performance of the final synthesized circulator (solid curves) and the performance of the initial circulator after the preliminary synthesis (dashed curves).

of the contour in $\phi/2 < \theta < \pi/3$ and computed by the equation

$$L = \int_{\phi/2}^{\pi/3} \frac{ds}{d\theta} d\theta \tag{6}$$

where $ds/d\theta$ is given by

$$\frac{ds}{d\theta} = \sqrt{\left(\frac{dr}{d\theta}\right)^2 + r^2}. \tag{7}$$

Therefore, the function F_2 is related to the extent of the pattern deformation of a six-sided resonator.

C. Algorithm for Optimization

As a mathematical technique for optimization, Powell's method, which is known as one of the direct search methods, is used because in our problem the derivatives of the evaluation function in terms of the variables are not obtained analytically. The method has been successfully used in the synthesis of a 3-dB hybrid planar circuit, which was the first synthesis of a practical planar circuit [7].

III. Results of Synthesis

In the numerical optimization of a wide-band circulator, the pattern of a resonator and a dielectric constant ϵ_s of the stripline coupled to the resonator are optimized for a specific material with a saturation magnetization $\mu_0 M_s$ of 0.13 T (1300 Gauss), and a dielectric constant ϵ_f of 15.6. The ferrite is dc biased in the direction perpendicular to the ground plane so that the internal magnetic field is zero [8]. Under such a bias condition, the lowest frequency in the operation range is located at 3.64 GHz ($= \gamma\mu_0 M_s$, here $\gamma = 2.8\times10^{10} s^{-1}T^{-1}$) to satisfy the condition that μ_{eff} is positive. All circuits are assumed to have the same thickness equal to 1.25 mm.

As an initial pattern for the synthesis, a six-sided resonator is used, which is optimized for wide-band operation in the preliminary synthesis. Therefore, in the actual synthesis, the synthesis process was divided into two parts.

A. Preliminary Synthesis of a Six-Sided Resonator Pattern

In this synthesis, only the three variables (such as the size, the coupling angle ϕ, and ϵ_s) are optimized according to the algorithm. The number of the total sampling points are 48, and 6 points are set along each port. The evaluation functions are computed at four frequencies of 4.0, 4.9, 5.9, and 6.6 GHz. The weighting parameters are chosen as $a_i = 1$ ($i = 1,\cdots,4$), $b_1 = b_2 = b_3 = 1$, $b_4 = 2$, and $c = 0$.

B. Final Synthesis

Ten sampling points are set on the periphery of one-sixth of the circuit to be synthesized, three of which are set along the straight part of the port. To describe an arbitrary shape, seven straight sections are used. Therefore, the number of variables used are nine: they are ϵ_s, ϕ, and seven distances between the center of the circuit and the sampling point on each straight section. In the computation of the evaluation function, four frequencies of 4.0, 5.0, 6.3, and 7.2 GHz are taken into account. The weighting parameters chosen in the synthesis are $a_1 = a_3 = a_4 = 1$, $a_2 = 3.7$, $b_1 = b_2 = b_3 = 1$, $b_4 = 2$, and $c = 1$, respectively. Only four courses of deformation were enough to obtain the final pattern. In one course, nine line searches corresponding to the variables and one subsequent pattern search are included.

The final obtained pattern and the six-sided pattern obtained in the preliminary synthesis for the one-sixth portion are shown in Fig. 2 in solid and dashed lines, respectively. The converged dielectric constants ϵ_s are 18.4 in the final synthesis and 21.0 in the preliminary synthesis. The computed performance of the circulators are shown in Fig. 3. The solid curves denote the performance of the finally synthesized circulator and the dashed curves are those after the preliminary synthesis. It is found that the performance is clearly improved in both bandwidth and loss characteristics in the final synthesis.

IV. Conclusion

As part of our effort in developing the microwave planar circuit theory, we have demonstrated a fully computer-aided synthesis of a planar circulator for wide-band operation. As a result, a circulator better than a six-sided (irregular hexagonal) circulator is synthesized. We believe that the preliminary synthesis of an initial pattern and the evaluation of pattern simplicity make our synthesis reasonable. The optimized circulator performance may be improved by adding external matching circuits to each circulator arm.

We hope the method of synthesis shown in this paper will be useful in the design of a circulator.

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On Mode Classification in Rectangular Waveguides Partially Filled with Dielectric Slabs

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Abstract —**Arguments are given for the designation of the lowest order longitudinal-section magnetic mode in a dielectrically loaded rectangular waveguide as LSM_{01} rather than LSM_{11}.**

Although the theory of dielectrically loaded rectangular waveguides is well established [1], [2], some inconsistency in the classification of the modes still remains. In particular, the lowest order longitudinal-section magnetic mode is designated as LSM_{11} [3], [4] instead of LSM_{01}. The present short paper provides arguments for the latter designation.

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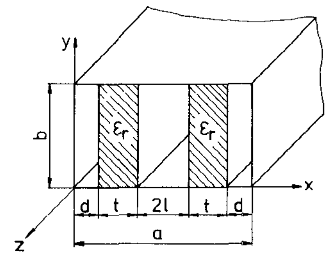

Fig. 1. A configuration under discussion.

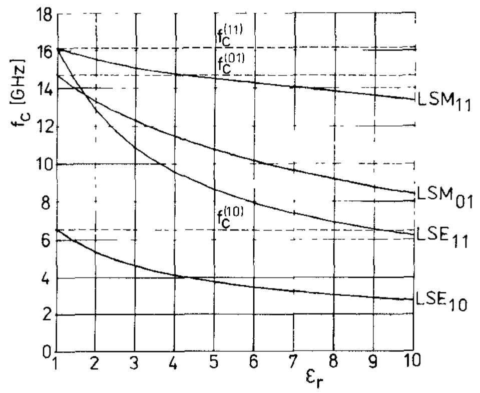

Fig. 2. The cutoff frequencies of several LSE and LSM modes in a waveguide with the dimensions $a = 22.86$ mm, $b = 10.16$ mm, $l = 0$, $2t = 6$ mm.

A typical and fairly general representative of the waveguides under discussion is shown in Fig. 1.[1] The guide can support longitudinal-section electric (LSE_{mn}) and longitudinal-section magnetic (LSM_{mn}) waves. The subscripts m and n are equal to the number of the field variations along the x and y axes, respectively. In an empty waveguide, the field can also be expressed in terms of LSE_{mn} and LSM_{mn} modes; these are, however, appropriate linear combinations of the commonly used TE_{mn} and TM_{mn} modes (transverse to the z axis) with the same subscripts m and n. It is therefore logical to require that the modes in a loaded waveguide be provided with subscripts m and n in such a manner that, approaching $\epsilon_r \to 1$, their field distribution becomes identical with that of equally designated modes in an empty guide. This "correspondence principle" is generally observed for the LSE modes. In a special case of no field variation along the y axis ($n = 0$), the LSE_{m0} modes have the electric field oriented in the y direction and approach the empty guide TE_{m0} modes (transverse to z).

However, it is as a rule not recognized [3], [4] that in the limit $\epsilon_r \to 1$, the lowest order LSM mode[2] becomes the TE_{01} (transverse to z) rather than the TM_{11} mode (a reason for this is probably intuitive connecting the lowest order LSM mode with the lowest order empty guide TM mode, i.e., TM_{11}). Hence, the LSM modes should be classified as

$$LSM_{mn}, \qquad m = 0,1,2,\cdots, \quad n = 1,2,3,\cdots.$$

It is beyond the scope of this paper to provide complete analysis, but a simple numerical example for a single centered slab ($l = 0$) readily confirms the above argument. In Fig. 2, cutoff frequencies

[1] Solution for a waveguide with one assymetrically placed slab, as well as for that with an H-plane slab, can be derived directly from the solution of the waveguide according to Fig. 1.

[2] This mode can be the dominant one for $a < b$ and ϵ_r near to unity.